

Passive Receiver Self-Localisation from Aircraft Bistatic Doppler Echoes and ADS-B Tracks

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Abstract

Global Navigation Satellite Systems (GNSS) are vulnerable to interference, motivating research into alternative positioning methods that exploit signals of opportunity. This study investigates whether an unknown receiver position can be estimated from bistatic Doppler measurements of aircraft-scattered signals when the transmitter position and aircraft trajectories are known. We formulate receiver localisation as an inverse bistatic Doppler problem in which each measured Doppler shift, combined with the corresponding aircraft position and velocity, defines an isodoppler constraint on the receiver position. The approach is evaluated using recordings of the French GRAVES space-surveillance radar, operating at 143.05 MHz, and aircraft trajectories obtained from the OpenSky Network. Doppler signatures are extracted from software-defined radio spectrograms, associated with individual aircraft, and combined to estimate the receiver position. We investigate the propagation of Doppler measurement uncertainty and the influence of transmitter–aircraft–receiver geometry through Monte Carlo simulations and a spatial geometry analysis. Of the 16 aircraft observed within the transmitter’s illumination sector, eight were retained for localisation, providing 160 isodoppler constraints. The resulting receiver-position estimate exhibited a horizontal error of approximately 26 km relative to the known reference position. The uncertainty analyses revealed pronounced directional dependence, with the degree of anisotropy varying across receiver locations. These findings provide an experimental demonstration of receiver localisation using aircraft-scattered bistatic Doppler measurements, while highlighting limitations in positioning accuracy and the importance of observation geometry. Further improvements in measurement accuracy and geometric diversity are required before the approach can be considered for practical positioning applications.

Keywords: passive bistatic radar; GRAVES; ADS-B; Doppler; receiver self-localisation

Abbreviations: ADS-B: Automatic Dependent Surveillance–Broadcast GRAVES: Grand Réseau pour la VEille Spatiale, SDR: Software-Defined Radio

1. Introduction

Global Navigation Satellite Systems (GNSS) provide positioning, navigation, and timing (PNT) information for a wide range of applications, from civil aviation and precision agriculture to the synchronisation of financial transactions. However, their low received signal power makes them vulnerable to jamming, spoofing, and other forms of radio-frequency interference [1, 2]. This vulnerability has motivated substantial research into alternative positioning, navigation, and timing (APNT) technologies. The United States Federal Aviation Administration (FAA) has investigated terrestrial alternatives for aviation, such as DME/DME-based positioning, since the early 2010s [3], and its navigation strategy calls for the continued development of resilient alternatives to GNSS [4]. In Europe, EUROCONTROL has similarly formalised the role of APNT for Performance Based Navigation (PBN) reversion [5]. More recently, Low-Earth-Orbit (LEO) satellite constellations have attracted growing

interest as a means of providing resilient PNT across a broad range of applications, with dedicated navigation systems such as the European Space Agency's (ESA) CELESTE [6] and Xona's Pulsar [7] under development.

In parallel, recent research has demonstrated the potential of *signals of opportunity* (SoOPs), i.e., radio-frequency transmissions not primarily intended for navigation that can nevertheless be exploited for PNT purposes. Relevant SoOPs include non-navigation satellite constellations, particularly LEO communication systems such as *Starlink*, terrestrial communication signals such as cellular [8] and Wi-Fi networks, and terrestrial broadcast signals such as FM radio and digital television [9, 10]. Depending on the signal, transmitter characteristics, and receiver architecture, different navigation observables can be extracted. These include timing-based observables such as time of arrival (TOA) and time difference of arrival (TDOA), as well as Doppler shift, carrier phase, angle of arrival, and received signal strength.

Of particular interest in this work are Doppler-based techniques, which exploit the frequency shift caused by relative motion between transmitter and receiver to obtain information about the relative velocity along the propagation path. Doppler-based positioning has a long history: the U.S. Navy's TRANSIT satellite navigation system, for example, estimated an unknown receiver position from the Doppler shift of signals transmitted by satellites with known orbits [11].

Doppler measurements can be obtained in monostatic, bistatic, or multistatic configurations. In a monostatic configuration, transmitter and receiver are co-located, whereas in a bistatic configuration they are spatially separated. A multistatic system combines measurements from multiple spatially distributed transmitters and/or receivers. Of particular relevance to this work is the bistatic case, in which a signal transmitted at one location is scattered by a moving target and observed at a spatially separated receiver. This configuration is commonly encountered in passive radar, where an existing non-cooperative transmitter serves as an *illuminator of opportunity* rather than the radar transmitting its own probing signal [12, 13].

In a bistatic configuration, the Doppler shift is governed by the change in the combined transmitter–target and target–receiver propagation path. For a transmitter at position $\vec{p}_T$, a moving target at $\vec{p}_A(t)$, and a receiver at $\vec{p}_R$, this bistatic path $R_b(t)$ is

$$R_b(t) = \|\vec{p}_A(t) - \vec{p}_T\| + \|\vec{p}_A(t) - \vec{p}_R\|. \quad (1)$$

The corresponding bistatic Doppler shift is proportional to the rate of change of this path,

$$f_D(t) = -\frac{1}{\lambda} \frac{dR_b(t)}{dt} = -\frac{1}{\lambda} \left[\underbrace{\vec{v}_A(t) \cdot \vec{u}_{TA}(t)}_{\text{transmitter side}} + \underbrace{\vec{v}_A(t) \cdot \vec{u}_{RA}(t)}_{\text{receiver side}} \right], \quad (2)$$

where λ is the carrier wavelength, $\vec{v}_A(t)$ is the target velocity, and $\vec{u}_{TA}(t)$ and $\vec{u}_{RA}(t)$ are unit vectors pointing from the transmitter and receiver, respectively, towards the target, as illustrated in Figure 1. Since $\vec{u}_{TA}(t)$ and $\vec{u}_{RA}(t)$ are unit vectors, the two dot products represent the projections of the target velocity onto the respective lines of sight. In particular, $\vec{v}_A(t) \cdot \vec{u}_{RA}(t) = \|\vec{v}_A(t)\| \cos \theta_R(t)$, where $\theta_R(t)$ denotes the angle between the target velocity and the receiver–target line of sight. The sign depends on the adopted Doppler convention. Consequently, the measured Doppler shift contains information about the target velocity as well as the transmitter–target–receiver geometry.

Aircraft have been extensively investigated as moving scatterers in passive bistatic radar. Howland, Maksimiuk, and Reitsma [14] demonstrated an experimental system using a single non-cooperative

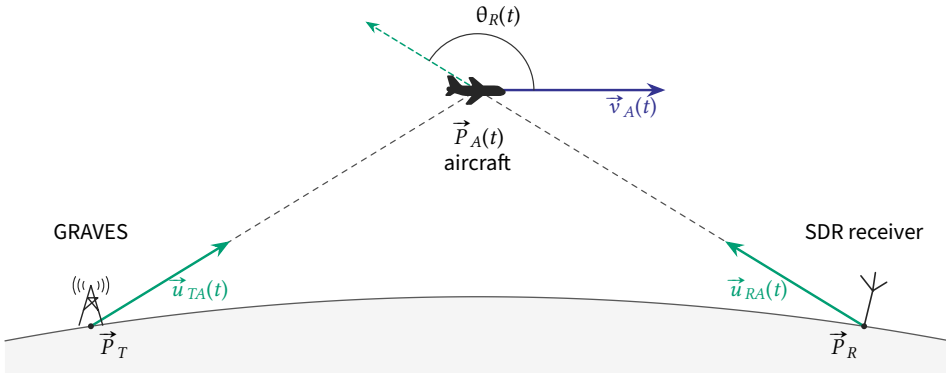


Figure 1. Bistatic transmitter-aircraft-receiver geometry. The unit vectors $\vec{u}_{TA}(t)$ and $\vec{u}_{RA}(t)$ point from the transmitter and receiver, respectively, towards the aircraft, while $\theta_R(t)$ denotes the angle between the aircraft velocity $\vec{v}_A(t)$ and the receiver-aircraft line of sight.

FM broadcast transmitter to detect and track aircraft at receiver ranges exceeding 150 km, with detections and tracks compared against a live civil air traffic control data feed [14]. Edrich, Schroeder, and Meyer [15] subsequently demonstrated a mature multi-band, multi-illuminator system exploiting FM, DAB, and DVB-T signals for aircraft detection, localisation, and tracking, while Moser et al. [16] showed that aircraft echoes from FM and DAB illuminators could be detected in real time using a low-cost Raspberry Pi and RTL-SDR receiver.

While these systems exploit Doppler together with range and, where available, bearing information, other work has investigated whether Doppler measurements alone are sufficient for aircraft tracking. Maasdorp, Inggs, and Nadjasngar [17] tracked a commercial aircraft using a single FM transmitter and Doppler measurements from four spatially distributed receivers, achieving position errors down to approximately 150 m. Conversely, Yu et al. [18] used multiple distant HF illuminators and a single receiver to estimate aircraft position and velocity from Doppler measurements using a Gaussian-mixture Bernoulli filter, with the resulting trajectory validated against ADS-B data.

A complementary line of research has considered the inverse positioning problem, in which the aircraft trajectory is known and used to determine the position of an unknown transmitter or receiver. Yi et al. [19] considered this problem in passive radar, using aircraft positions obtained from ADS-B together with bistatic range measurements to determine the position of an unknown transmitter. Eichelberger et al. [20] instead considered an unknown receiver and used ADS-B and Mode S messages as GPS-like ranging signals: reference ground stations estimate the transmission times of aircraft messages, after which the receiver position is obtained from their measured arrival times using multilateration. Canals et al. [21] proposed a signal-matching approach to reduce the required observation time. Using ADS-B messages decoded by distributed reference receivers, they reconstructed the transmitted waveforms and searched for the receiver position for which their predicted arrival times best matched a short raw SDR recording, requiring only three seconds of observations. More recently, Lizarribar et al. [22] developed *SkyPos*, a reverse-multilateration approach that compares the arrival times of the same ADS-B messages at the unknown receiver and GPS-synchronised OpenSky reference receivers to estimate the receiver position.

Taken together, these studies demonstrate that known aircraft trajectories can provide geometric information for determining unknown transmitter or receiver positions. Existing approaches to receiver localisation, however, exploit signals transmitted directly by the aircraft, whereas passive-

radar approaches using aircraft with known trajectories have considered bistatic range measurements to determine an unknown transmitter position. To the best of our knowledge, an unknown receiver position has not previously been determined from bistatic Doppler measurements of signals scattered by aircraft with known trajectories. In this work, we formulate receiver localisation as an inverse bistatic Doppler problem. The illuminator is a non-cooperative transmitter at a known position $\vec{p}_T$, while the position $\vec{p}_A(t)$ and velocity $\vec{v}_A(t)$ of the scattering aircraft are obtained from ADS-B trajectory data provided through the OpenSky Network [23]. The receiver position $\vec{p}_R$ is unknown and enters directly into the bistatic Doppler relationship above. Each aircraft state and corresponding Doppler measurement defines an isodoppler constraint on $\vec{p}_R$, which in the two-dimensional localisation problem considered here corresponds to an isodoppler curve. Measurements collected along an aircraft trajectory therefore provide a family of such curves, which can be combined across multiple aircraft and observation geometries to estimate the receiver position.

Against this background, this work examines whether an unknown receiver position can be determined from bistatic Doppler measurements associated with known aircraft trajectories, how uncertainty in the extracted Doppler measurements propagates into receiver-position uncertainty, the extent to which the transmitter–aircraft–receiver geometry influences its directional characteristics, and what positioning accuracy can be achieved using real-world measurements.

We investigate these questions using the French *Grand Réseau pour la Veille Spatiale* (GRAVES) space-surveillance radar as the illuminator of opportunity. Developed by the French Aerospace Research Centre (ONERA), GRAVES employs a bistatic continuous-wave (CW) radar architecture operating in the very-high-frequency (VHF) band and uses Doppler measurements for the detection of orbiting objects [24]. Its continuous-wave signal is particularly well suited to the proposed Doppler-based approach: since the transmitted waveform is an unmodulated continuous carrier, an aircraft echo appears as a frequency-shifted component relative to the carrier, allowing its bistatic Doppler shift to be extracted directly from an SDR spectrogram. Unlike passive-radar systems exploiting modulated illuminators such as FM or DAB, this does not require reconstruction or cross-correlation of the transmitted waveform to obtain the Doppler measurement [13, 16]. The experimental evaluation combines SDR recordings of aircraft-scattered GRAVES signals with aircraft trajectories obtained from the OpenSky Network, treating the known receiver position as unknown during localisation and using it only for validation.

The paper makes four contributions. First, we develop a processing chain that extracts aircraft Doppler signatures from SDR spectrograms and associates them with ADS-B trajectories. Second, we formulate the isodoppler constraint for the inverse bistatic Doppler localisation problem and use the resulting curves to estimate the unknown receiver position. Third, we investigate the propagation of Doppler measurement uncertainty into receiver-position uncertainty and characterise the influence of transmitter–aircraft–receiver geometry on uncertainty anisotropy. Finally, we evaluate the approach experimentally using real GRAVES recordings and quantify the resulting receiver-positioning accuracy.

2. Methods

The methodology comprises the experimental setup and data acquisition, Doppler measurement extraction and aircraft association, inverse bistatic Doppler modelling, receiver-position estimation, and the analysis of position uncertainty and observation geometry.

2.1 Experimental Setup and Data Sources

As indicated in Figure 1, the experimental setup consists of the GRAVES transmitter serving as a source of signal of opportunity, an SDR-based receiver at a known reference location, and aircraft

acting as moving scatterers between the transmitter and receiver. Although the receiver position $\vec{p}_R$ is known for validation purposes, it is treated as unknown during localisation. The GRAVES transmitter is located in Broye-lès-Pesmes, near Dijon (47.3481°N, 5.5151°E), France, and operates at a carrier frequency of 143.05 MHz. The GRAVES signal was received using a *Nooelec RTL2832U* RTL-SDR receiver with the telescopic whip antenna supplied with the receiver, and recorded using the *Gqrx* software [25]. The receiver was tuned to 143.048 MHz, which is 2 kHz below the GRAVES carrier frequency, and the signal was demodulated in upper-sideband (USB) mode. This offset serves two purposes: it moves the GRAVES carrier away from the DC spike of the SDR, and it places the carrier at an audio frequency of approximately 2 kHz, so that aircraft echoes with both positive and negative Doppler shifts appear at positive audio frequencies. The demodulated audio signal was saved as a WAV file, allowing Doppler-shifted aircraft echoes to be identified in its spectrogram. The SDR recording used in this study was obtained on 14 December 2025 and covers the period from 06:36:17 to 07:28:17 UTC.

Aircraft trajectories were obtained from the OpenSky Network [23] and processed using the *trafic* library [26]. OpenSky data were retrieved for 14 December 2025 between 06:00 and 08:00 UTC within a 100 NM bounding box around the GRAVES transmitter. Observations below 10 000 ft were discarded, after which the trajectories were resampled at 1 s intervals. To retain aircraft within the relevant GRAVES observation geometry, a ground-projected illumination sector spanning azimuth angles from 90° to 270° was defined. Its radial bounds were obtained from the ground distance $d = h / \tan \epsilon$, using 20 000 ft (FL200) at an elevation angle of 40° for the inner boundary and 40 000 ft (FL400) at 15° for the outer boundary. An aircraft observation was classified as being within the illumination sector if its bearing from the GRAVES transmitter was between 90° and 270° and its horizontal distance from the transmitter was between the inner and outer radial boundaries. Contiguous sequences of observations within the illumination sector were grouped into illumination intervals. Each interval began with the first in-sector observation and ended at the first subsequent out-of-sector observation; if the trajectory ended while the aircraft remained inside the sector, the final observation time was used as the interval end. Illumination intervals belonging to the same aircraft and separated by gaps of at most 10 s were subsequently merged. The trajectories were then restricted to the SDR measurement period and truncated to these illumination intervals. Resulting trajectory segments shorter than 10 s were excluded. For each retained aircraft, the trajectory provides the time-dependent aircraft position $\vec{p}_A(t)$ and velocity $\vec{v}_A(t)$, which are subsequently used as known aircraft states for Doppler-ridge association and receiver localisation.

Figure 2 illustrates the nominal aircraft illumination footprint and potential ground receiver locations for Mode-S reception. The coverage model uses the trajectory-selection mask described above, with aircraft at FL 370 for the terrain-visibility calculation.

2.2 Doppler Measurement Extraction and Aircraft Association

For each retained aircraft trajectory, the corresponding illumination interval determined in Section 2.1 was selected from the SDR recording. The signal within this interval was transformed into a time–frequency representation using a short-time Fourier transform with 8192-sample windows and 50 % overlap. The analysis was restricted to the frequency range from 1900 Hz to 2125 Hz, containing the observed aircraft-scattered GRAVES signal. To suppress stationary spectral components, the spectral power was logarithmically transformed and its temporal median was subtracted separately at each frequency bin. A moving aircraft produces a time-varying Doppler-shifted component in the resulting spectrogram, appearing as a curved spectral trace hereafter referred to as the Doppler ridge. Frequencies in the recorded spectrogram are expressed relative to the receiver tuning and therefore include a constant frequency offset f_{off} . The physical bistatic Doppler shift used in the

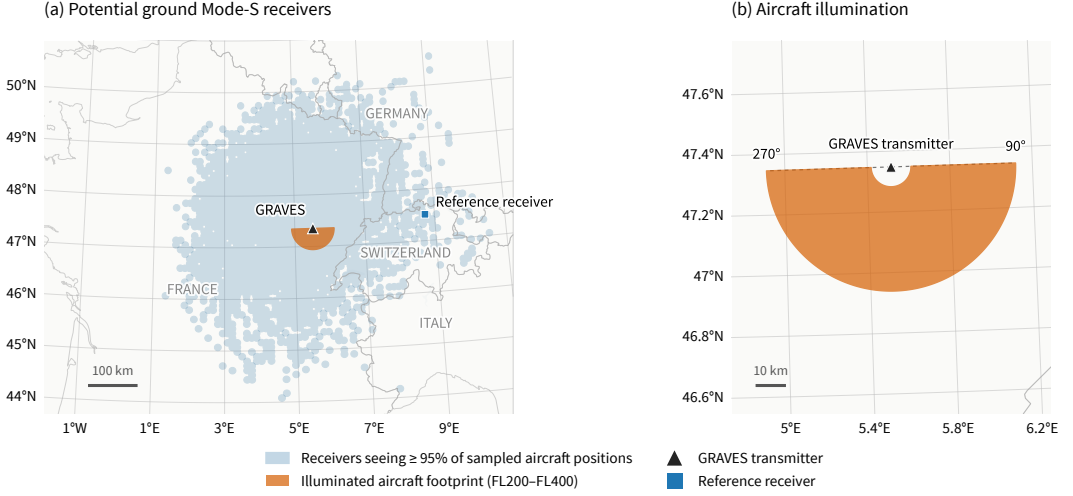


Figure 2. Illustrative GRAVES aircraft illumination and ground receiver visibility. (a) Shaded locations have modelled line of sight to at least 95 % of sampled aircraft positions, accounting for SRTM terrain, an aircraft altitude of FL 370 and an effective Earth-radius factor of 4/3. (b) Ground-projected aircraft-selection footprint for FL 200–FL 400, spanning azimuths from 90° to 270° with elevation bounds of 15° to 40°. Visibility is geometric and does not include a radio link budget.

subsequent localisation is obtained as

$$f_D(t) = f_{\text{ridge}}(t) - f_{\text{off}}, \quad (3)$$

where $f_{\text{ridge}}(t)$ denotes the frequency of the fitted ridge in the spectrogram and $f_{\text{off}} = 2035 \text{ Hz}$ is the constant frequency offset associated with the SDR recording.

The association of a Doppler ridge with a particular aircraft was performed using its known trajectory. According to Equation (2), the transmitter-side Doppler contribution $\vec{v}_A(t) \cdot \vec{u}_{TA}(t)$ can be calculated directly from the known transmitter position and aircraft state. In contrast, the receiver-side contribution $\vec{v}_A(t) \cdot \vec{u}_{RA}(t)$ cannot be evaluated because $\vec{p}_R$ is unknown. Geometrically, this contribution is the projection of the aircraft velocity onto the receiver–aircraft line of sight and can therefore be written as $\|\vec{v}_A(t)\| \cdot \cos \theta_R(t)$, where $\theta_R(t)$ denotes the angle between the aircraft velocity vector and $\vec{u}_{RA}(t)$, as shown in Figure 1. Over the observation interval of an individual aircraft, the temporal evolution of the geometric angle $\theta_R(t)$ was approximated by the quadratic function

$$\hat{\theta}_R(t) = \frac{A}{10^4} t + B + \frac{C}{10^8} t^2, \quad (4)$$

where $\hat{\theta}_R(t)$ denotes the approximation of $\theta_R(t)$, A , B , and C are fitted coefficients, and the factors 10^4 and 10^8 scale the linear and quadratic terms during optimisation. Combining this approximation with the known transmitter-side contribution in Equation (2) provides a family of candidate Doppler ridges for each aircraft trajectory without requiring prior knowledge of the receiver position.

The coefficients were estimated by selecting the candidate ridge that maximised the accumulated spectrogram energy along its path over the aircraft observation interval. An initial two-dimensional grid search was performed over $A \in [-15, 15]$ and $B \in [-1.5, 1.5]$, followed by joint optimisation of A , B , and C using the covariance matrix adaptation evolution strategy (CMA-ES) [27], with $C \in [-150, 150]$. The fitting was performed independently for each aircraft over its corresponding GRAVES illumination interval. When multiple spectral ridges were present simultaneously, candi-

date curves could intersect or follow different spectral features; the optimisation selected the parameter combination yielding the greatest accumulated spectral energy while remaining constrained by the Doppler evolution associated with the considered aircraft trajectory. The fitted ridge is thereby associated with that aircraft, and Equation (3) provides the corresponding physical Doppler measurements used for receiver localisation.

2.3 Inverse Bistatic Doppler Model

The fitted Doppler ridge provides a time-dependent Doppler measurement $f_D(t)$ along the aircraft trajectory. To construct individual isodoppler constraints, the ridge was evaluated at discrete observation times t_i . The corresponding Doppler measurement $f_D(t_i)$ is associated with the aircraft position $\vec{p}_A(t_i)$ and velocity $\vec{v}_A(t_i)$. For each observation time t_i , Equation (2) can be rearranged to isolate the receiver-side contribution,

$$\underbrace{\vec{v}_A(t_i) \cdot \frac{\vec{p}_A(t_i) - \vec{p}_R}{\|\vec{p}_A(t_i) - \vec{p}_R\|}}_{\text{receiver-side contribution}} = -\lambda f_D(t_i) - \underbrace{\vec{v}_A(t_i) \cdot \vec{u}_{TA}(t_i)}_{\text{transmitter-side contribution}}. \quad (5)$$

The right-hand side contains only measured or known quantities, whereas the left-hand side depends on the unknown receiver position $\vec{p}_R$. Consequently, each Doppler measurement defines a set of receiver positions satisfying Equation (5). This set constitutes an *isodoppler curve*: all receiver positions along the curve produce the same Doppler shift for the given aircraft state.

Receiver localisation was formulated as a two-dimensional problem with known receiver altitude, which was fixed to 470 m. Equation (5) was evaluated in Earth-centred, Earth-fixed (ECEF) coordinates. Aircraft positions and candidate receiver positions, expressed in geodetic longitude, latitude, and altitude, were transformed to ECEF coordinates before evaluating the geometric terms of the equation. Isodoppler curves were constructed within a geographical search region spanning 4° to 10°E and 46° to 48.5°N . This restriction also excludes potential alternative solutions associated with geographically separated intersections of the isodoppler constraints. At each sampled longitude, the residual of Equation (5) was evaluated as a function of latitude, and a zero crossing was determined using Brent's root-finding method [28]. Repeating this procedure across the longitude range yielded a sequence of geographical coordinates satisfying the Doppler constraint, which together formed the isodoppler curve for observation time t_i .

2.4 Receiver Position Estimation

As the aircraft moves, Doppler measurements obtained at different observation times t_i define different isodoppler curves. For each aircraft, the fitted Doppler ridge was evaluated at $N = 20$ observation times distributed uniformly between 25 % and 75 % of the retained observation interval to avoid boundary effects, yielding a set of isodoppler curves $\{\mathcal{I}_i\}_{i=1}^N$. These curves provide multiple geometric constraints on the same receiver position.

Let $d(\vec{p}_R, \mathcal{I}_i)$ denote the minimum Euclidean point-to-polyline distance between a candidate receiver position $\vec{p}_R$ and isodoppler curve $\mathcal{I}_i$, evaluated in longitude–latitude coordinates and expressed in degrees.

$$\hat{\vec{p}}_R = \arg \min_{\vec{p}_R} \sum_{i=1}^N (d(\vec{p}_R, \mathcal{I}_i))^2. \quad (6)$$

In the absence of measurement and modelling errors, the isodoppler curves would intersect at the receiver position. In practice, Equation (6) determines the position that best reconciles the non-intersecting constraints.

The consistency of each aircraft-specific solution was quantified by the root-mean-square distance between the estimated receiver position and its N isodoppler curves,

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^N \left(d\left(\hat{\vec{p}}_R, \mathcal{I}_i\right) \right)^2}. \quad (7)$$

Aircraft-specific solutions with an RMSE greater than 0.015° or an estimated receiver position outside the geographical plausibility region spanning 5.5° to 10.5°E and 46° to 48.5°N were excluded from further processing.

The isodoppler curves of all retained aircraft were subsequently pooled into a common set and Equation (6) was solved again to obtain the *combined receiver-position estimate*. All retained isodoppler constraints were weighted equally. Two objective functions were considered: the L2 objective, minimised using the L-BFGS-B algorithm, and the L1 objective, minimised using Powell's derivative-free method to improve robustness against outlier trajectories. Both optimisation methods were implemented using *SciPy* [29], with the candidate receiver position constrained to the geographical search region spanning 4° to 10°E and 46° to 48.5°N . The optimisation was initialised at the mean longitude and latitude of all points comprising the isodoppler curves.

Localisation accuracy was evaluated using the known receiver position $\vec{p}_R^{\text{ref}}$, which was used only as ground truth and was not provided to the estimation procedure. Unlike the point-to-polyline distances used for receiver-position estimation, the horizontal position error was calculated as the geodesic distance between the estimated and reference receiver coordinates,

$$e_R = d_{\text{geo}}\left(\hat{\vec{p}}_R, \vec{p}_R^{\text{ref}}\right), \quad (8)$$

where $d_{\text{geo}}(\cdot, \cdot)$ denotes the geodesic distance between two geographical positions.

2.5 Position-Solution Uncertainty and Geometric Analysis

The accuracy and observability of the receiver-position solution depend on both uncertainties in the quantities entering the bistatic Doppler model and the transmitter–aircraft–receiver geometry. These effects were investigated through an uncertainty-propagation analysis and a spatial analysis of the observation geometry.

2.5.1 Uncertainty Propagation

Uncertainty in the extracted Doppler ridges propagates through the isodoppler constraints to the estimated receiver position. This effect was quantified using a Monte Carlo analysis with 100 realisations, in which the fitted ridge-parameter vector $\beta = [A, B, C]^\top$ was randomly perturbed. For each realisation, independent zero-mean Gaussian perturbations with a standard deviation of 0.025 were added to the three ridge parameters. The perturbed Doppler ridge was subsequently used to reconstruct the corresponding isodoppler curves and repeat the receiver-position estimation described in Section 2.4. This procedure was applied both to individual-aircraft solutions and to the combined multi-aircraft solution.

For the combined solution, the ridge parameters of all retained aircraft were perturbed in each Monte Carlo realisation, after which their resulting isodoppler curves were pooled and a new combined receiver position was estimated. The set of aircraft was kept fixed throughout the analysis; no re-sampling of aircraft was performed. The resulting receiver-position samples therefore quantify the propagation of uncertainty in the extracted Doppler ridges through the localisation procedure.

The distribution of receiver-position samples was characterised by its sample mean and covariance matrix Σ_R . A 95 % uncertainty ellipse was obtained from the covariance matrix using the corresponding two-dimensional chi-squared quantile $\chi^2_{2,0.95} = 5.991$. The eigenvectors of Σ_R determine the principal directions of the ellipse, while its major and minor semi-axis lengths a and b are given by

$$a = \sqrt{\chi^2_{2,0.95} \lambda_1}, \quad b = \sqrt{\chi^2_{2,0.95} \lambda_2}, \quad \lambda_1 \geq \lambda_2, \quad (9)$$

where λ_1 and λ_2 are the eigenvalues of Σ_R . The anisotropy of the position solution was characterised by the major-to-minor semi-axis ratio

$$\rho = \frac{a}{b} = \sqrt{\frac{\lambda_1}{\lambda_2}}, \quad (10)$$

with values close to unity indicating approximately isotropic position uncertainty and larger values indicating an increasingly elongated solution.

2.5.2 Spatial Geometry Sweep

To investigate the spatial dependence of the receiver localisation geometry, hypothetical receiver positions were selected using Latin hypercube sampling (LHS). A total of 980 candidate receiver locations were initially considered, of which 330 were retained based on their visibility of aircraft within the GRAVES illumination footprint. To determine the suitable receiver locations, aircraft positions were sampled within the illumination footprint between FL 300 and FL 400, and receiver-to-aircraft line of sight was evaluated at a representative aircraft altitude of FL 350. The visibility analysis accounted for terrain elevation based on SRTM data, a receiver antenna height of 10 m above ground level, and Earth curvature and atmospheric refraction using an effective Earth-radius factor of 4/3. Receiver locations with line of sight to at least 95 % of the sampled aircraft positions were retained. These locations were subsequently used to define the geographical region considered in the spatial geometry sweep.

At each of the 330 selected receiver locations, the receiver altitude was fixed to 470 m, while the GRAVES transmitter position and observed aircraft trajectories were retained. The expected bistatic Doppler shifts were calculated from Equation (2), thereby generating synthetic Doppler observations corresponding to the transmitter-aircraft-receiver geometry at each hypothetical receiver location.

For each hypothetical receiver position, a parametric Doppler ridge was fitted to the theoretical Doppler evolution of each of the 16 available aircraft by minimising the mean squared error (MSE). Aircraft for which the resulting root mean squared error (RMSE) did not exceed 2 Hz were retained for the subsequent bootstrap analysis. Bootstrap resampling was performed at the aircraft-trajectory level, meaning that entire aircraft trajectories, rather than individual Doppler observations, were re-sampled with replacement. The size of each bootstrap sample corresponded to the number of valid aircraft trajectories at the respective receiver location, which ranged from 13 to 16. For each bootstrap sample, the receiver position was estimated, resulting in a distribution of position estimates from which a confidence ellipse was computed.

The spatial variation in the directional characteristics of the position uncertainty was quantified using the major-to-minor uncertainty-axis ratio ρ defined in Equation (10). The axis ratios obtained from the bootstrap analysis at the sampled receiver positions were calculated in a local east-north metric coordinate system and spatially interpolated using piecewise cubic interpolation, as implemented in `scipy.interpolate.griddata` with `method="cubic"`, to generate an empirical anisotropy map. Values of ρ close to unity indicate geometries that constrain the receiver similarly in both

horizontal directions, whereas larger values indicate increasingly anisotropic geometries. This analysis permits the geometry at the experimental receiver location to be interpreted relative to the surrounding region.

In addition to the empirical analysis, the directional characteristics of the receiver localisation geometry were evaluated theoretically using the local sensitivity of the bistatic Doppler shift to changes in receiver position. For each hypothetical receiver position, the Doppler Jacobians were calculated along the trajectories of all 16 aircraft and combined to construct an information matrix. The corresponding covariance approximation was then used to determine the major-to-minor uncertainty-axis ratio ρ , with the calculations performed in a local east–north metric coordinate system to ensure comparability with the empirical analysis. In contrast to the empirical anisotropy map, the theoretical anisotropy was evaluated directly at each point of a regular geographic grid comprising 240 by 180 points. The resulting axis ratios were visualised using filled contours without applying scattered-data interpolation.

3. Results

The results are presented in three parts: Doppler measurement extraction and aircraft association, receiver-position uncertainty and its spatial variation, and experimental receiver localisation.

3.1 Doppler Measurement Extraction and Aircraft Association

Within the SDR observation period (06:36:17 to 07:28:17 UTC on 14 December 2025), a total of 16 aircraft trajectories were observed within the illumination sector of GRAVES. Of these, seven were excluded based on the quality of the extracted Doppler ridges, and one was excluded because the corresponding receiver-position estimate fell outside the predefined geographical search region. The remaining eight aircraft were retained for the combined receiver-position estimation using the procedure described in Sections 2.2 and 2.4.

Figure 3 shows a representative processed spectrogram for *EasyJet Europe* flight EJU16MB, together with the fitted Doppler ridge (solid blue line) and the theoretical Doppler evolution (dark dashed line). The theoretical Doppler curve was calculated using Equation (2), based on the known transmitter and reference receiver positions and the aircraft trajectory. Over the selected observation interval, the fitted ridge extends from approximately 2090 Hz to 1905 Hz. The root mean squared error (RMSE) and mean absolute error (MAE) between the fitted and theoretical Doppler curves were 2.70 Hz and 1.76 Hz, respectively.

3.2 Position-Solution Uncertainty and Geometric Analysis

Receiver-position uncertainty was evaluated through Monte Carlo simulations, while the spatial variation in uncertainty anisotropy was examined using the geometry sweep.

3.2.1 Uncertainty Propagation

Figure 4 shows the receiver-position estimates obtained from 100 Monte Carlo realisations for flight EJU16MB_165. The resulting estimates form an elongated distribution oriented approximately southwest–northeast, with a corresponding 95 % covariance ellipse. The figure also shows the mean and maximum a posteriori (MAP) estimates alongside the known reference receiver position.

For the combined multi-aircraft solution, the 95 % uncertainty ellipse had major and minor semi-axis lengths of 107.76 km and 3.88 km, respectively, corresponding to an axis ratio of $\rho = 27.78$. The major axis was oriented 8.29° counterclockwise from east, while the area enclosed by the ellipse was approximately 1313 km^2 . The resulting ellipse therefore indicates a pronounced directional depen-

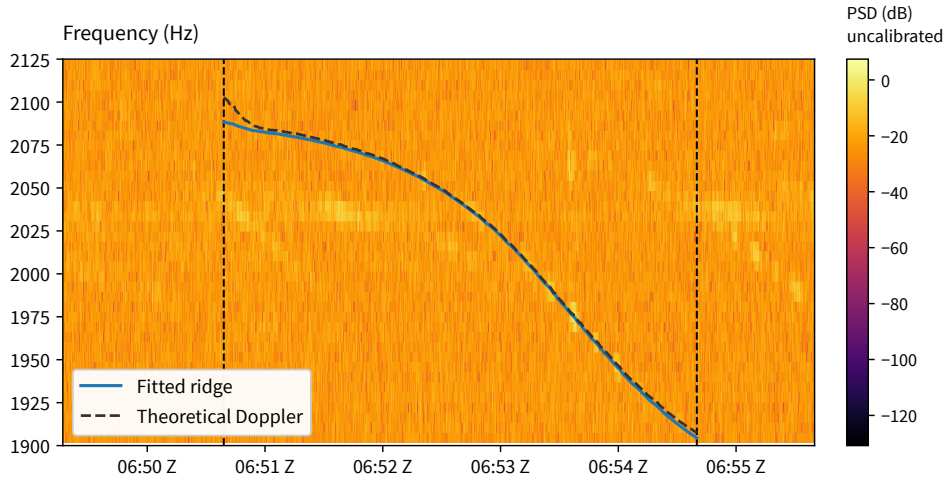


Figure 3. Processed spectrogram for flight EJU16MB_165 with the fitted Doppler ridge and theoretical Doppler evolution. The vertical dashed lines indicate the observation interval used for ridge fitting.

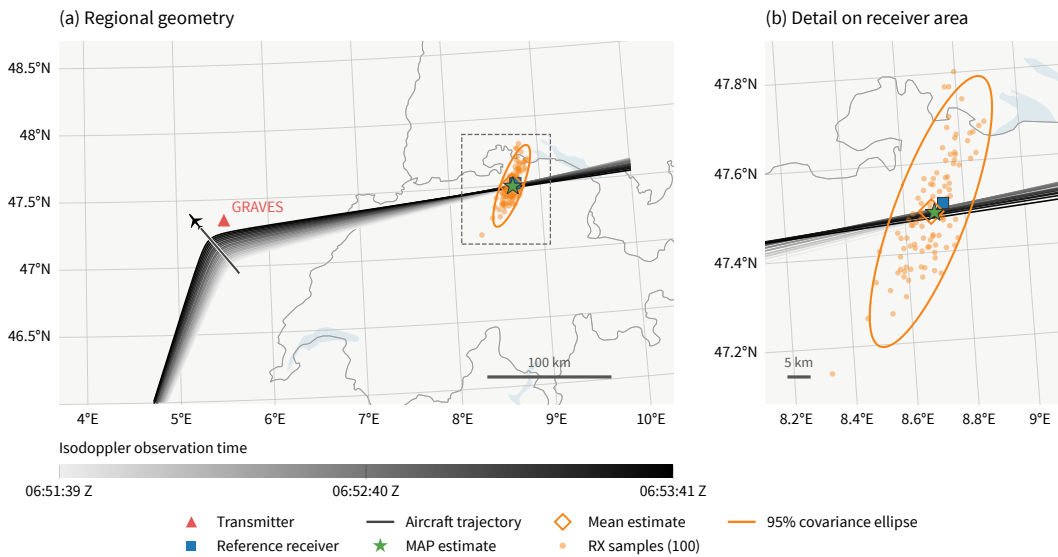


Figure 4. Receiver-position uncertainty for flight EJU16MB_165. (a) Regional geometry showing the aircraft trajectory and isodoppler constraints. (b) Detailed view of the 100 Monte Carlo receiver-position estimates, their mean, the maximum a posteriori (MAP) estimate, the reference receiver position, and the 95 % covariance ellipse.

dence of the receiver-position uncertainty, with considerably greater uncertainty along the major axis than in the perpendicular direction.

3.2.2 Spatial Geometry Sweep

Figure 5 shows the spatial distribution of empirical and theoretical receiver-position uncertainty anisotropy, expressed as the major-to-minor axis ratio ρ . The empirical anisotropy in Figure 5(a) exhibits lower axis ratios across much of the central and western evaluation region, with higher values concentrated towards the eastern boundary, particularly around 9°E and 47°N. A smaller region of elevated anisotropy is also visible in the northern part of the evaluation area. The theoretical anisotropy in Figure 5(b) shows a smoother spatial distribution, with lower ratios in the central region and increasing values towards the western and eastern boundaries.

Across the evaluated receiver positions, the empirical axis ratio ranged from 1.38 to 65.43, while the theoretical axis ratio ranged from 3.19 to 29.45. At the experimental receiver location, the empirical and theoretical axis ratios were $\rho = 47.40$ and $\rho = 19.39$, respectively. Although the two approaches differ in their absolute values and spatial distributions, both indicate that the receiver-position uncertainty depends strongly on the observation geometry.

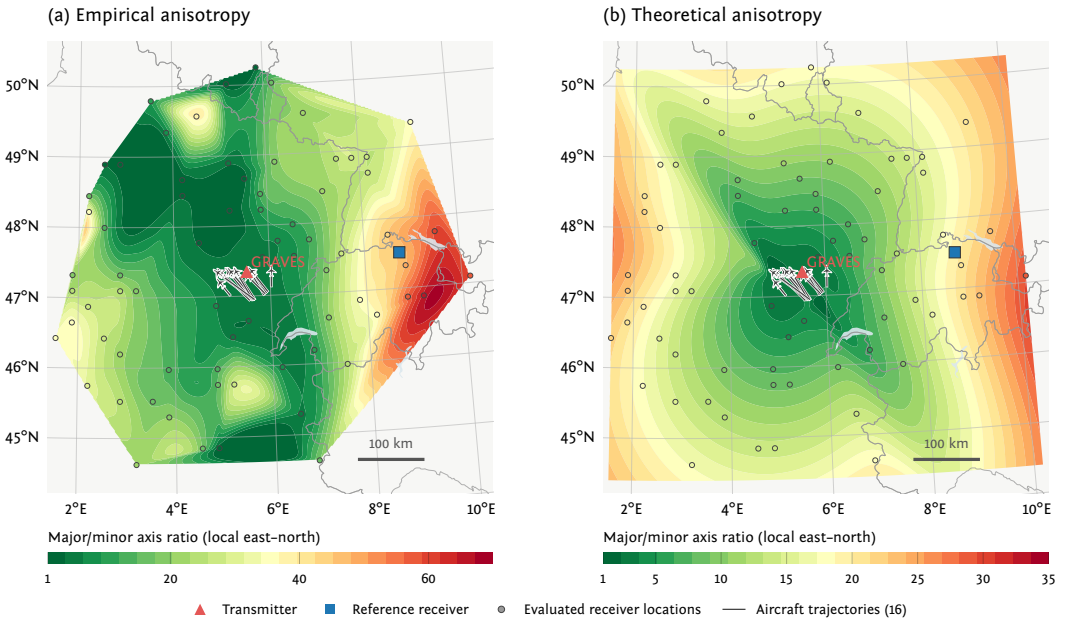


Figure 5. Receiver-position uncertainty anisotropy, expressed as the major-to-minor axis ratio ρ : (a) empirical estimates from covariance ellipses of bootstrap receiver positions at sampled locations, spatially interpolated using piecewise cubic interpolation; (b) Doppler-Jacobian estimates evaluated directly on a regular grid. Panels use separate linear colour scales.

3.3 Experimental Receiver Localisation

Following the aircraft-specific RMSE and geographical plausibility filtering described in Section 2.4, 8 aircraft were retained for the combined receiver-position estimate. With $N = 20$ isodoppler curves per aircraft, the combined solution was therefore based on 160 isodoppler constraints. Figure 6 shows the retained aircraft-specific position estimates and isodoppler curves together with the resulting combined receiver-position estimate and the known reference receiver position.

The combined solution obtained by minimising the L1 loss yielded an estimated receiver position

of 8.392°E , 47.458°N , with an RMSE of 0.01386° according to Equation (7). The corresponding horizontal position error relative to the known reference receiver position was approximately 26.04 km. Additionally, the combined solution obtained by minimising the L2 loss in (6) yielded an estimated receiver position of 8.274°E , 47.450°N , with an RMSE of 0.0185° according to Equation (7). The corresponding horizontal position error relative to the known reference receiver position was approximately 34.92 km.

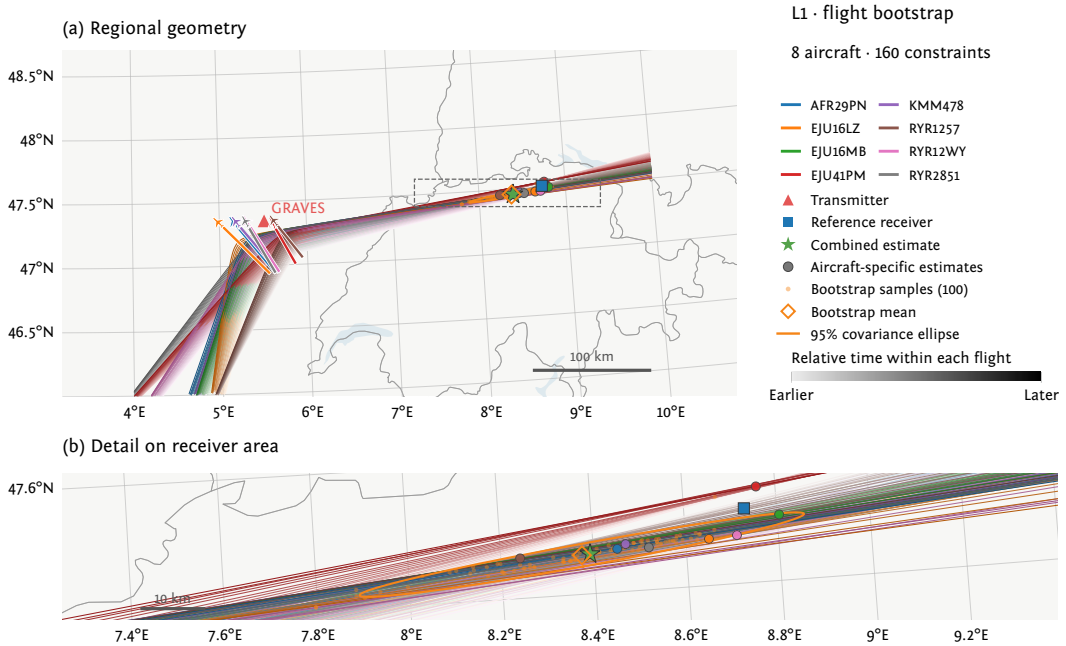


Figure 6. Receiver-position estimate for the combined multi-aircraft solution using 160 isodoppler constraints from eight aircraft. The L1 solution has a horizontal position error of approximately 26.04 km. The covariance ellipse summarises the spread of 100 whole-flight bootstrap estimates.

Notably, flights EJU41PM and RYR1257, corresponding to the two easternmost outlier trajectories, appear to contribute disproportionately to the receiver-position estimation error. Figure 7 examines the sensitivity of the combined solution to the loss function and the inclusion of EJU41PM and RYR1257. For each aircraft selection, 100 bootstrap realisations were generated by sampling entire aircraft trajectories with replacement, retaining all 20 isodoppler constraints for each sampled trajectory. Identical resamples were used for the L1 and L2 solutions within each selection. Position errors were evaluated in a local east–north coordinates system. Using all eight aircraft, the L1 and L2 estimates had local horizontal errors of 26.04 km and 34.92 km, respectively. Excluding the two aircraft reduced these errors to 13.34 km and 16.83 km, while also reducing the bootstrap spread. L2 produced narrower covariance ellipses, whereas L1 yielded smaller errors for the estimates obtained from each complete selection, illustrating the distinction between precision and accuracy. The distributions remained elongated and displaced from the reference position. This exclusion experiment is a sensitivity analysis, and the improved agreement with the reference does not, by itself, justify rejecting either aircraft.

4. Discussion

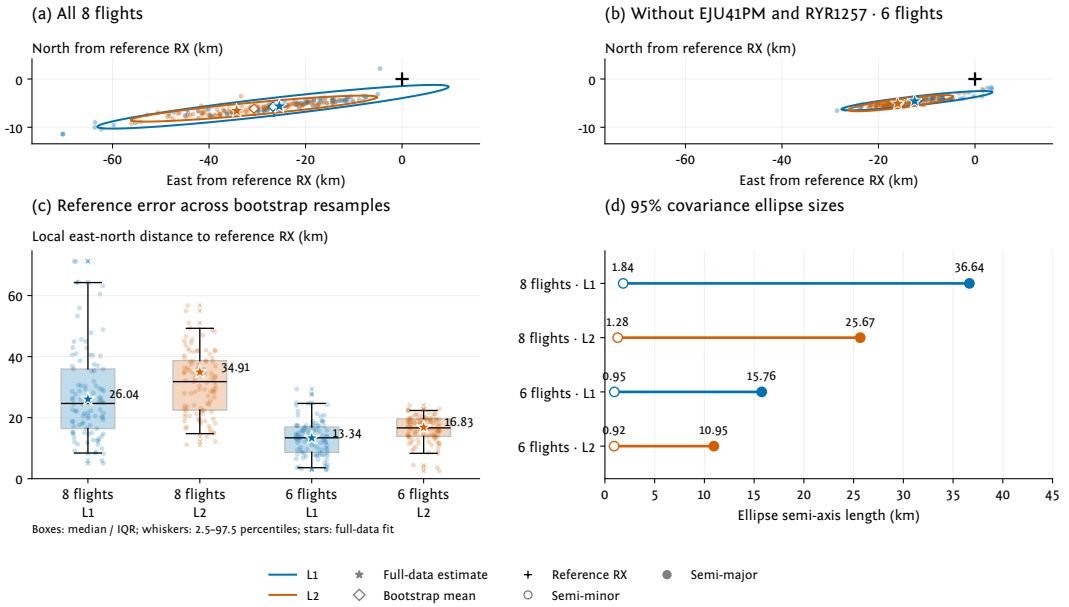


Figure 7. Combined receiver-position estimates using L1 and L2 losses with all eight aircraft and excluding EJU41PM and RYR1257. Panels (a,b) show 100 whole-flight bootstrap estimates and 95% covariance ellipses in local east-north coordinates. Stars, diamonds, and the black cross indicate full-selection estimates, bootstrap means, and the reference receiver, respectively. Panels (c,d) compare horizontal error distributions and ellipse semi-axis lengths.

4.1 Principal Findings and Research Questions

The results demonstrate the feasibility of estimating an unknown receiver position from bistatic Doppler measurements of aircraft-scattered signals when the transmitter position and aircraft trajectories are known. Using GRAVES as a non-cooperative illuminator and aircraft trajectories obtained from the OpenSky Network, Doppler signatures were extracted from SDR recordings and associated with individual aircraft. The resulting isodoppler constraints were combined across eight aircraft to obtain a receiver-position estimate without using the known receiver location during estimation. These findings provide an experimental demonstration of the inverse bistatic Doppler localisation principle.

The uncertainty analysis revealed a pronounced directional dependence of receiver-position uncertainty. The Monte Carlo analysis demonstrated that perturbations of the fitted Doppler-ridge parameters propagated into an elongated distribution of receiver-position estimates, indicating that the available observations constrained the receiver position more strongly in some directions than in others. For the combined multi-aircraft solution, the 95 % uncertainty ellipse had major and minor semi-axis lengths of 107.76 km and 3.88 km, respectively, corresponding to an axis ratio of $\rho = 27.78$. The spatial geometry sweep further showed that this anisotropy varies with the hypothetical receiver location. At the experimental receiver location, the empirical and theoretical analyses yielded axis ratios of $\rho = 47.40$ and $\rho = 19.39$, respectively. Although the two approaches differed in their spatial distributions and magnitudes, both indicated a pronounced dependence of the receiver-position uncertainty on the observation geometry. These findings are consistent with the broader observation by Yi et al. [19] that the geometry of aircraft observations influences localisation uncertainty, although their analysis concerns transmitter localisation using bistatic range measurements rather than receiver localisation using bistatic Doppler measurements. Taken together, these findings highlight the importance of transmitter-aircraft-receiver geometry in determining the di-

rectional characteristics of receiver-position uncertainty.

An important implication of these findings is that aircraft trajectories provide not only the information required to interpret the measured Doppler shifts, but also determine the geometric constraints available for receiver localisation. Unlike approaches that exploit signals transmitted directly by aircraft [20, 21, 22], the proposed method uses aircraft as moving scatterers of a signal emitted by a separate, fixed transmitter. Consequently, the localisation geometry depends on the relative positions and motion of all three components: transmitter, aircraft, and receiver. The observed spatial variation in uncertainty anisotropy suggests that the selection of aircraft observations could play an important role in improving localisation performance. In particular, combining observations with complementary geometries may help reduce directional uncertainty and improve the conditioning of the position-estimation problem.

4.2 Positioning Performance and Navigation Potential

Although the experimental results demonstrate that receiver localisation is possible using aircraft-scattered bistatic Doppler measurements, the achieved positioning accuracy remains limited. The estimated receiver position differed from the known reference position by approximately 26 km, indicating a substantial discrepancy despite the successful extraction and combination of Doppler observations from eight aircraft. Previous experimental studies demonstrated substantially higher positioning accuracy using Doppler-only passive radar, including the aircraft-tracking experiments of Maasdorp, Inggs, and Nadjiasngar [17] and Yu et al. [18]. However, these studies estimate the position of a moving aircraft using known radar configurations, whereas the present work estimates an unknown receiver position using known aircraft trajectories. Their results are therefore not directly comparable, but demonstrate that Doppler measurements can provide useful positioning information under suitable observation geometries. The uncertainty analysis further indicates that the combined receiver-position solution is considerably better constrained in one horizontal direction than in the perpendicular direction, as reflected by the large axis ratio of the corresponding uncertainty ellipse. Consequently, the positioning performance cannot be adequately characterised by a single horizontal error value alone. The sensitivity analysis further illustrates the distinction between positioning precision and accuracy: although L1 yielded smaller horizontal positioning errors, L2 produced less dispersed bootstrap estimates. Excluding two aircraft trajectories reduced the positioning errors of both estimators, but this improvement does not establish that the excluded observations were erroneous or justify their removal from the primary analysis.

With the present configuration (a single illuminator, a low-cost receiver with a whip antenna, and a limited number of suitably illuminated aircraft), the achieved accuracy is far from sufficient for any practical positioning or navigation application. The principle is nevertheless of interest because it exploits signals from an existing non-cooperative transmitter, with aircraft acting as moving scatterers rather than dedicated navigation beacons. However, the method requires knowledge of both the transmitter position and the aircraft trajectories. In the present implementation, the latter are obtained from ADS-B reports, whose position information is generally derived from GNSS. Consequently, the demonstrated approach cannot currently be considered independent of GNSS. Furthermore, its practical availability depends on the presence of suitable aircraft, their illumination by the transmitter, and the ability to extract sufficiently reliable Doppler measurements. Whether the principle could eventually contribute to APNT therefore remains open: this would require not only substantially improved positioning accuracy, but also an assessment of availability, integrity, continuity, and positioning latency.

4.3 Limitations

The experimental evaluation is based on a single recording of approximately 52 min, obtained at one receiver location using one non-cooperative transmitter. Although Doppler signatures from multiple aircraft were identified, only eight aircraft were ultimately retained for receiver localisation. The findings therefore reflect a limited set of aircraft trajectories and observation geometries under the conditions prevailing during this particular experiment. In particular, the results do not establish whether comparable localisation performance can be achieved at other receiver locations, with different aircraft traffic patterns, or under different signal reception conditions. Further experimental campaigns would be necessary to assess the reproducibility of the results and the extent to which the proposed approach generalises to other observation scenarios. Furthermore, the geographical search constraints exclude potential alternative solutions outside the predefined region, meaning that global uniqueness of the receiver-position estimate has not been established.

A further limitation concerns the accuracy of the Doppler measurements and the aircraft trajectories used for localisation, both of which are important considerations in Doppler-based passive-radar positioning [17, 18]. The Doppler observations are obtained from spectral ridges extracted from SDR recordings and subsequently approximated by a parametric model. Errors in ridge extraction, imperfect agreement between the fitted model and the measured Doppler signatures, and uncertainties in the assumed frequency offset may therefore affect the resulting isodoppler constraints. A more frequency-stable SDR and an antenna providing improved signal reception could potentially reduce measurement errors and increase the number of usable aircraft echoes, although their effect on positioning accuracy was not investigated. Similarly, the aircraft positions and velocities obtained from ADS-B reports are subject to measurement and reconstruction errors, which propagate into the predicted bistatic Doppler shifts. The localisation model also assumes that the observed signals can be adequately described by the bistatic propagation geometry involving the transmitter, aircraft, and receiver. Deviations from these assumptions may introduce systematic errors that are not necessarily captured by the uncertainty analysis.

Finally, the uncertainty analyses are subject to methodological limitations. The Monte Carlo analysis propagates prescribed perturbations of the fitted Doppler-ridge parameters into receiver-position estimates, rather than accounting for all potential sources of measurement and modelling error. Similarly, the empirical bootstrap analysis characterises the variability of position estimates under resampling of the available aircraft trajectories, but does not necessarily capture systematic errors shared across observations. The theoretical Doppler-Jacobian analysis provides a complementary assessment of geometric sensitivity based on a local linearisation of the measurement model, consistent with the broader use of information-based uncertainty analysis in passive-radar localisation [19]. Consequently, the resulting uncertainty estimates depend on their respective assumptions and should not be interpreted as comprehensive bounds on positioning error. In particular, they cannot establish whether the estimated receiver position is unbiased or explain the full discrepancy between the estimated and known receiver locations. Furthermore, the empirical and theoretical anisotropy analyses yielded different axis ratios at the experimental receiver location. While both approaches indicate a pronounced directional dependence of receiver-position uncertainty, the empirical analysis reflects the variability of position estimates under resampling of aircraft trajectories, whereas the theoretical analysis characterises the local sensitivity of the Doppler measurement model. The observed differences should therefore be interpreted in the context of the assumptions underlying the respective analyses.

5. Conclusion and Outlook

This study investigated the feasibility of estimating an unknown receiver position from bistatic Doppler measurements of aircraft-scattered signals. Using GRAVES as a non-cooperative transmitter and aircraft trajectories obtained from the OpenSky Network, we developed and experimentally evaluated a localisation approach in which known aircraft positions and velocities are combined with measured Doppler signatures to constrain the receiver position. Unlike conventional passive-radar approaches that estimate the positions of moving targets, the proposed method treats the aircraft trajectories as known and the receiver position as the unknown quantity.

The experimental evaluation demonstrated that Doppler observations from eight aircraft could be combined to estimate the receiver position, yielding a horizontal positioning error of approximately 26 km relative to the known reference location. While this accuracy is far too low for any practical application in the present configuration, the results constitute a proof of principle: aircraft-scattered bistatic Doppler measurements contain exploitable information on the receiver position, and this information can be extracted from real-world recordings. The Monte Carlo uncertainty analysis further revealed a pronounced directional dependence of receiver-position uncertainty, with a major-to-minor uncertainty-axis ratio of $\rho = 27.78$ for the combined multi-aircraft solution. The spatial geometry sweep demonstrated that this anisotropy varied considerably with the transmitter-aircraft-receiver geometry, highlighting the importance of selecting aircraft observations that provide complementary geometric constraints.

Future research should focus on improving the accuracy and robustness of the proposed localisation approach. Particular attention should be given to identifying and mitigating systematic errors in Doppler extraction and modelling, improving the accuracy of aircraft trajectory information, and investigating strategies for selecting aircraft observations with complementary geometries. Additional measurement campaigns involving different receiver locations, transmitters, and traffic conditions would help establish the reproducibility and generalisability of the results. Finally, assessing the availability and reliability of suitable Doppler observations, together with the potential use of aircraft trajectory information independent of GNSS, would be necessary to evaluate the approach as a possible complementary positioning source for APNT applications.

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Author Contributions

Conceptualization (J.D.-C., M.F., X.O., M.W.), Methodology (J.D.-C.), Software (J.D.-C., M.F.), Validation (J.D.-C., M.W., X.O.), Data Curation (V.F., M.F.), Writing – Original Draft (M.W.), Writing – Review & Editing (*all*), Visualization (X.O., J.D.-C.)

Open Data and Reproducibility Statement

The code used to retrieve and process the OpenSky aircraft trajectory data and to generate the results presented in this study is publicly available at <https://github.com/ZHAW-ZAV/osn26-graves>. The raw SDR recording used in the experimental evaluation is available on Zenodo at <https://zenodo.org/records/23559067>.

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